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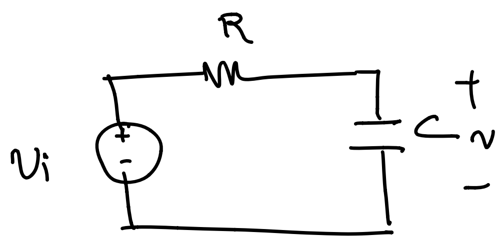

$v_i = V_m \cos(\omega t)$

$$v_i = RC\frac{dv}{dt} + v$$

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{V_m}{RC}\cos(\omega t)$$

Now just math

Find integrating factor

$$e^{\int \frac{1}{RC}dt} = e^{t/RC}$$

$$e^{t/RC}\frac{dv}{dt} + \frac{1}{RC}v\,e^{t/RC} = e^{t/RC}\frac{V_m}{RC}\cos(\omega t)$$

$$\int \frac{d}{dt}\left[e^{t/RC}v\right] = \int e^{t/RC}\frac{V_m}{RC}\cos(\omega t)$$

$$e^{t/\tau}v = \frac{V_m}{\tau}\int e^{t/\tau}\cos(\omega t)$$

$$e^{t/\tau} v = \frac{\frac{V_m}{\tau}\left[e^{t/\tau} \omega \sin(\omega t) + \frac{e^{t/\tau}}{\tau} \cos(\omega t)\right]}{\frac{1}{\tau^2} + \omega^2} + K_1$$

$$v = K_1 e^{-t/\tau} + \frac{V_m}{\tau\left(\frac{1}{\tau^2}+\omega^2\right) e^{t/\tau}}$$

$$\boxed{Y = A\cos\omega t + B\sin\omega t \doteq Y = \sqrt{A^2+B^2}\cos\left(\omega t + \tan^{-1}\left(\frac{-B}{a}\right)\right)}$$

$$A = \frac{e^{t/\tau}}{\tau}$$

$$B = e^{t/\tau}\omega$$

$$\left[\sqrt{\frac{e^{2t/\tau}}{\tau^2} + e^{2t/\tau}\omega^2}\cos(\omega t + \phi)\right.$$

$$e^{t/\tau}\sqrt{\left(\frac{1}{\tau^2}+\omega^2\right)}\cos(\omega t + \phi)$$

$$\tan^{-1}\left[-\frac{\cancel{e^{t/\tau}}\omega}{\frac{\cancel{e^{t/\tau}}}{\tau}}\right]$$

$$\tan^{-1}(-\tau\omega)$$

$$v = K_1 e^{-t/\tau} + \frac{V_m\, \cancel{e^{t/\tau}}\sqrt{\left(\frac{1}{\tau^2}+\omega^2\right)}\cos(\omega t + \phi)}{\tau\left(\frac{1}{\tau^2}+\omega^2\right)\cancel{e^{t/\tau}}}$$

$$v = K_1 e^{-t/\tau} + \frac{V_m \cos(\omega t + \phi)}{\tau\sqrt{\frac{1}{\tau^2} + \omega^2}}$$

$$\sqrt{1 + \omega^2 \tau^2}$$

$$v = K_1 e^{-t/\tau} + \frac{V_m \cos(\omega t + \phi)}{\tau\sqrt{\frac{1 + \omega^2\tau^2}{\tau^2}}}$$

$$\boxed{v = K_1 e^{-t/\tau} + \frac{V_m \cos(\omega t + \phi)}{\sqrt{1 + \omega^2 \tau^2}}}$$

Transient + steadystate

$$v_C = K_1 e^{-t/RC} + \frac{\cos(2\pi f t + \phi)}{\sqrt{1 + (2\pi f RC)^2}}$$

$$\phi = \tan^{-1}(-2\pi f RC)$$

Appendix:

why integrating factor works?

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$e^{\int P(x)\,dx}\frac{dy}{dx} + e^{\int P(x)\,dx}P(x)y = f(x)$$

$$\frac{d}{dx}\left(e^{\int P(x)\,dx} \cdot y\right)$$ check